

Exercise 40

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$$

Solution

Make the substitution, $u = \ln x$. Then as $x \rightarrow 0^+$, $u \rightarrow -\infty$.

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x) = \lim_{u \rightarrow -\infty} \tan^{-1} u$$

Make another substitution, $v = \tan^{-1} u$, or $\tan v = u$. Then as $u \rightarrow -\infty$, $v \rightarrow -\frac{\pi}{2}^+$.

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x) = \lim_{v \rightarrow -\frac{\pi}{2}^+} v$$

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x) = -\frac{\pi}{2}$$